A MATHEMATICAL MODEL OF THE SPROCKET - TENSIONER AND/OR ROAD – WHEEL - HUB MOTOR/GENERATOR FOR THE INTELLIGENT MAIN BATTLE TANK

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Abstract

In the paper has been formulated a mathematical model of the sprocket-, tensioner- and/or road-wheel-hub motor/generator for the electrically-powered and mechatronically-controlled intelligent main battle tank (IMBT). In order to formulate a mathematical model of the sprocket-, tensioner- and/or road-wheel-hub AC-AC, AC-DC-AC or DC-AC/AC-DC macrocommutator magnetoelectrically-excited in-wheel-hub motor/generator, analogous to the mathematical model of the brushed DC-AC/AC-DC commutator IPM magnetoelectrically-excited motor/generator with a rotating DC-AC/AC-DC mechanoelectrical commutator taking into consideration equations of unholonomic constraints of the AC-AC, AC-DC-AC or DC-AC/AC-DC commutator, a set of the second-order Euler-Lagrange differential equations of dynamics in a matrix notation for the AC commutatorless on-board generator can be written. After taking into account the equations of unholonomic constraints of the MCT or MOSFET AC-DC/DC-AC macrocommutator in the differential equations of dynamics one can be obtained differential equations of dynamics, establishing of a mathematical model of AC-DC/DC-AC macrocommutator magnetoelectrically excited in-wheel-hub generator/motor with the MCT or MOSFET application specific integrated matrixer (ASIM) AC-DC/DC-AC macrocommutator, acting as the electrical machine's MCT or MOSFET ASIM AC-DC rectifier/DC-AC inverter.

Keywords: *mathematical model, physical model, main battle tank (MBT), motor, generator, electric wheel*

1. Introduction

In October 1979, one of the greatest scientists in the history of 'terramechanics', the never-enough-to-be-regretted Pole Professor Dr M.G. ('Greg') Bekker of the U.S.A.[1], 'Father of Terrain-Vehicle Systems', best known for the invention of a steel transparent-type caterpillar track and pioneering construction of a steel wire-mesh wheel-tyre and other members for lunar rowing vehicle (LRV) for the National Aeronautics and Space Administration (NASA), asked the author to develop an all-automatic axleless continuously variable transmission (ACVT) for civilian and military tracked all-terrain vehicle suitable for mass use (inexpensive and easily manufactured) that would improve internal combustion engine (ICE) or external combustion engine (ECE) performance and vehicle driveability and breakability when operated by any (even unskilled) human driver (HD).

Tracked all-terrain vehicles generally have distinct advantages over wheeled all-terrain vehicles on cross-country terrain, primarily because the caterpillar track results in a lower ground pressure, leading to reduced sinkage and a better distribution of tractive effort.

When the road wheels (RW) are connected by a caterpillar track, their motion is not independent, and this, together with the motion of the caterpillar track itself, makes prediction much more difficult. Bearing in mind the sophistication and high cost of such tracked all-terrain vehicles, this difficulty is a serious handicap for the automotive scientists and engineers. This is particularly true of fighting vehicles which additionally usually have quite complex active absorb-by-wire (ABW) all-wheel-absorbed (AWA) suspension mechatronic control systems.

By contrast the motion of the caterpillar track or *'chain'* of modern unconventional electrically-powered and mechatronically-controlled tracked all-terrain vehicles is very difficult and much more sophisticated because:

- sprocket wheel (SW) torque is applied through a sprocket motorized and/or generatorized wheel (SM&GW) as in conventional tracked all-terrain vehicles, but additionally auxiliary road wheel (RW) torques are applied through road motorized and/or generatorized wheels (RM&GW) and auxiliary tensioner wheel (TW) torque is applied through a tensioner motorized and/or generatorized wheel (TM&GW), which all moves with the vehicle;
- the load is distributed in an interminate manner along a finite length of essentially stationary caterpillar track in contrast with the ground;
- the caterpillar-track inertia is a significant function of the automotive functional hypersystem inertia, and its motion is very non-linear;
- there is significant coulomb friction between adjacent caterpillar-track links.

The author started his research and development (R&D) work on mono- and/or poly-drive electromechanical drive-by-wire (DBW) all-wheel-driven (AWD)propulsion as well as mono-and/or poly-brake mechanical-electrical brake-by-wire (BBW) all-wheel-braked (AWB) dispulsion mechatronic control systems in the early 1980s [2]. His developments were directed towards the automotive industry but his mono- and/or poly-drive electromechanical DBW propulsion as well as mono- and/or poly-brake mechanic-electrical BBW dispulsion spheres also had some industrial applications.

The author boast of yourself as a pioneer in the application of electromechanical DBW AWD propulsion and mechanic-electrical BBW AWB dispulsion mechatronic control systems to civilian and military, wheeled and tracked, on/off-road vehicles. As early as 1982, the author introduced electromechanical GBW AWD propulsion and mechanic-electrical BBW AWB dispulsion mechatronic control systems to wheeled automotive vehicles [2, 3], and as early as 1984 - for wheeled and tracked all-terrain vehicles [3, 4]. The author envisages a time when fully-automatic motion control of fighting vehicles will be standard equipment on all main battle tanks (MBT). In the nearest future, the intelligent main battle tank (IMBT) is expected to make a big contribution to the society composed largely of unskilled HDs. It will be of great help to the unskilled soldiers, vastly improvimg their quality of fighting. Moreover, it also will be useful for HDs who are not used to MBTs. For this reason, the author is pursuing R&D work in this field.

2. Experimental Proof-of-Concept DBW AWD Propulsion and BBW AWB Dispulsion

The experimental proof-of-concept DBW AWD propulsion and BBW AWB dispulsion, mecha-tronic control systems used in high-speed, terrestrial and water IMBTs satisfy nearly all the same essential requirements as for the running gear systems used on high-speed military tracked all-terrain vehicles, namely:

(i) to apply an all-electric-wheel DBW propulsion sphere to a large number of RM&GWs,
(ii) to apply an all-electric-wheel BBW dispulsion sphere to a large number of RM&GWs,
(iii) to allow a large ABW AWA suspension displacement to allow high-speed motion over cross-country difficult terrain,
(iv) to allow at the outer side of the curve a positive propelling (driving) torque and at the inner side - a negative dispelling (braking) torque, achieving their maximum value for pivot skid steering,
(v) to occupy the minimum volume within the space envelope of the IMBT,
(vi) to distribute the mass of the IMBT over a relatively large ground surface or soil area.

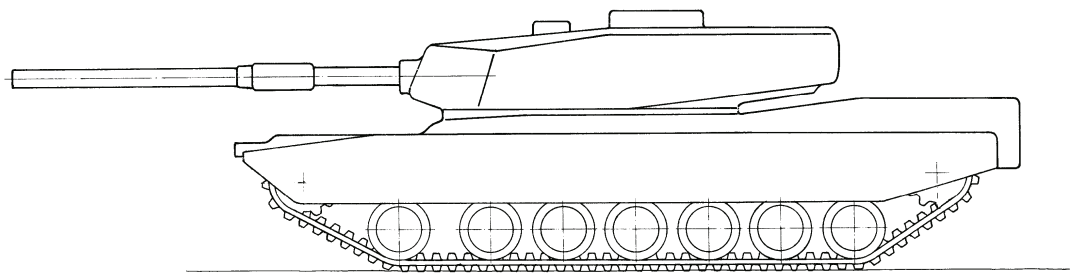
The requirements (i) and (ii) will contribute to the very good soft soil performance of IMBTs. The feature (v) will tend to comflict with (i) and (ii).

The DBW AWD propulsion and BBW AWB dispulsion mechatronic control systems must be of minimum mass, reliable and durable as well as easy to maintain and, compared to some other MBT components, reasonably inexpensive to manufacture.

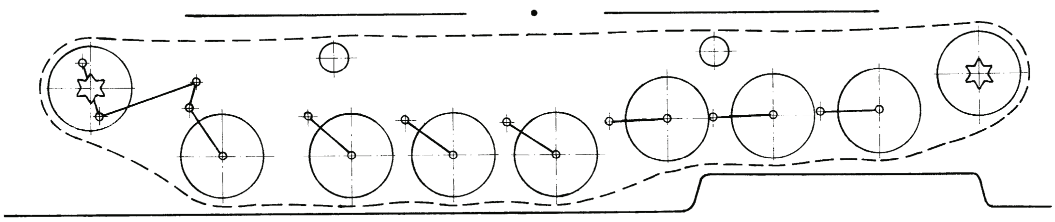
In the fighting-vehicle field an example of study is represented by experimental proof-of-concept series hybrid chemo-thermo-fluid-mechanic-electromechanical and/or electromechanical DBW AWD propulsion as well as mechanoelectrical BBW AWB dispulsion mechatronic control systems for an electrically-powered and mechatronically-controlled IMBT with extremely high mobility and steerability, which is conceived by the author at Cracow University of Technology, Cracow, Poland.

The experimental proof-of-concept series hybrid-electric chemo-thermo-fluid-mechanic-electro-mechanical and/or electromechanical DBW AWD propulsion and mechanoelectrical BBW AWB dispulsion are designed for installation in the hypothetical, high-speed, terrestrial and water, PL Pulaski IMBT armed with a light anti-tank gun and/or uranium arrow missile with loading automat. It will have a turret that is a strong and hard thick active heavily armoured metal-powder structure protecting a gunner or crew (two men only), made so as to revolve with the gun and/or rocket. It will also have very small windows that withstand sledgehammer blows, a hull and road-wheel tyres that endure multiple shots from a firearm, and a rampart that sprays tear gas.

Figure 1 shows an overall view of the PL Pulaski IMBT and its caterpillar-track loop for a 65 km/h traversal of a 0.25 m high obstacle. The barycentre is located at the centre of the hull.



(Top)



(Bottom)

Fig. 1. The layout (Top) of the PL Pulaski IMBT and its caterpillar- track loop (Bottom) for a 65 km/h traversal of a 0.25 m high obstacle

The seven RM&DWs on each side of the hull have independent rotary-damper ABW AWA suspensions. The front SM&GWs (sprockets) and the rear TM&GWs as well as fourteen (seven on each side) RM&GWs are driven individually by the AC-AC or DC-AC/AC-DC macro-commutator in-wheel-hub motors/generators, respectively, and the rotating speed of each DBW×BBW caterpillar track and each SM&GWs, TM&GWs and RM&GWs can be arbitrarily controlled by a driver-vehicle and terrain-vehicle real-time expert hypersystem, incorporating model following fuzzy-logic programmable and neural-network learning motion control of an IMBT.

3. Major Components of DBW AWD Propulsion and BBW AWB Dispulsion

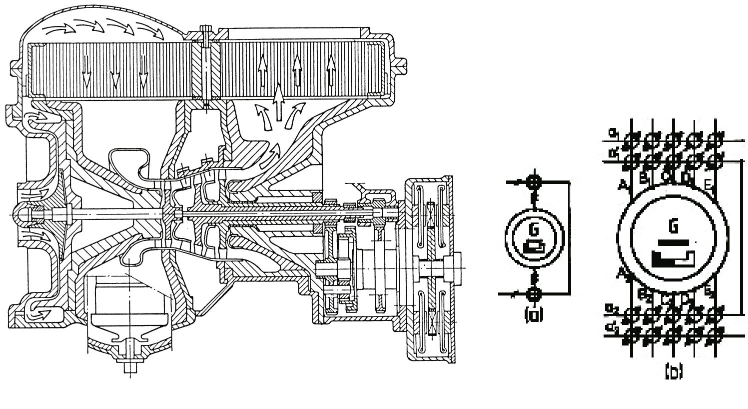
Major components that determine the tractive performance of the proof-of-concept series hybrid-electric chemo-thermo-fluid-mechanic-electromechanical DBW AWD propulsion and mechanic-electrical BBW AWB dispulsion mechatronic control systems for the IMBT are:

- two automotive gas turbine-generators/motors (AGT-G/M) that are based on the Fijalkowski turbine boosting (FTB)system,
- two DBW × BBW caterpillar tracks incorporating two SM&GWs and two TM&GWWs as well as fourteen RM&GWs with eighteen brushless type DC-AC/AC-DC macrocom-mutator in-wheel-hub motors/generators,
- an on-board artificial intelligence (AI) application specific integrated circuit (ASIC) fuzzy-logic and neural-network (FN) microcomputer-based fuzzy-logic programmable and neural-network learning (P&L) controller, i.e., AI ASIC FN microcontroller.

A. Automotive Gas Turbine-Generator/Motor (AGT-G/M)

After intensive R&D work of various turbine transmission and component arrangement [...], the AGT-G/M that is based on the FTB system with a component arrangement as shown in Figure 2 (Left) was chosen for the experimental mechanically power boosted AGT-G/Ms.

The heart of the FTB system is a primary-energy-source mechanical energy-storing high-speed AGT-G/M incorporating the brushless type AC commutatorless or AC-DC/DC-AC macro-commutator [2 × 5-phase wire-wound (slotted-core) disc-shaped stators and unwound interior permanent magnet (IPM) disc-shaped rotor] magnetoelectrically-excited composite-flywheel- disc generator/motor. The arrangement is very simple as shown in Figure 2.



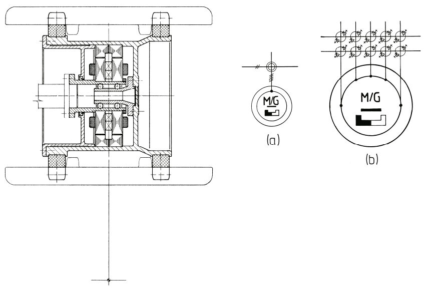
(Left) (Right)

Fig. 2. The layout (left) of the automotive as turbine-generator/motor (AGT-G/M) and its (right) brushless type AC commutatorless or AC-DC/DC-AC macrocommutator magnetoelectrically-excited composite-flywheel-disc generator/motor

B. Sprocket-, Tensioner- and Road Wheels (SM&GWs, TM&GWs and RM&GWs) with the AC-AC or DC-AC/AC-DC macrocommutator magnetoelectrically-excited and reluctance in-wheel-hub motor/generator

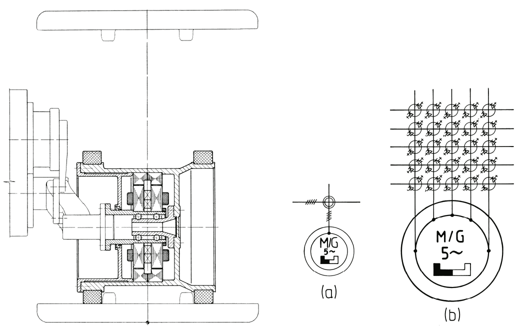
The SM&GWs, TM&GWs and RM&GWs are fitted with the brushless AC-AC or DC-AC/AC-DC macrocommutator [5-phase wire-wound and unwound IPM inner stator and unwound mild-soft iron (MSI) rotor] magnetoelectrically-excited and reluctance in-wheel-hub motors/generators, respectively. The rotating housing is made in the form of sprocket-, tensioner- and road-wheel hubs.

As a result of the application specific integrated matrixer (ASIM) AC-AC or DC-AC/AC-DC macrocommutator design principle [1] of the sprocket-, tensioner- and road-wheel hub motors/generators, the following important features are obtained: full starting torque directly in the SM&GWs, TM&GWs and RM&GWs without the loss of efficiency caused by intermediate gearing; superior smoothness in torque and/or speed control through outstanding low-speed tractive performance; no mechanical axles - complete freedom in the design of the new concept DBW propulsion and BBW dispulsion mechatronic control systems; high-external loads; constant torque throughout the full rotation; mechanic-electrical regenerative coasting and braking as well as skid-steering; reversibility; free-wheeling; and extremely low-noise level (especially valid during the DBW × BBW all-electric operation on the battle fields). Figure 3 shows the principle layout of a mechatronically-controlled planetary-gearless SM&GW or TM&GW with the DC-AC/AC-DC macrocommutator [5-phase wire-wound and unwound IPM inner stator and unwound MSI outer rotor] magnetoelectrically-excited and reluctance in-wheel-hub motor/generator, and Figure 4 - the principle layout of a mechatronically-controlled planetary-gearless RM&GW with the AC-AC macrocommutator [5-phase wire-wound and unwound IPM inner stator and unwound MSI outer rotor] magnetoelectrically-excited and reluctance in-wheel-hub motor/ generator, all conceived and newly designed by the author.



(Left) (Right)

Fig. 3. The layout (left) of the sprocket and/or tensioner motorized and/or generatorized (SM&GW & TM&GW) and its (right) brushless type AC commutatorless or AC-DC/DC-AC macrocommutator magnetoelectrically-excited and reluctance in-wheel-hub motor/generator



(Left) (Right)

Fig. 4. The layout (left) of the road motorized and/or generatorized (RM&GW) and its (right) brushless type AC commutatorless or AC-DC/DC-AC macrocommutator magnetoelectrically-excited and reluctance in-wheel-hub motor/generator

The mechatronically-controlled planetary-gearless SM&GWs, TM&GWs and RM&GWs with the AC-AC or DC-AC/AC-DC macrocommutator magnetoelectrically-excited and reluctance sprocket- or tensioner-wheel-hub motors/generators thoroughly eliminate backlashes or hysteresis, which are inevitable in using any speed reducers.

It is evident that there are no windings on the MSI outer rotor, no slip-rings, and no mechanic-electrical commutator with the sliding copper segments and their carbon brushes.

The nature of the inner-stator 5-phase armature winding is also not evident. If the inner stator is a six-pole, 5-phase structure with a large number of 'teeth', then the outer rotor is also toothed and magtnetized so that a south magnet pole occupies one-half of the peripheries while a north magnet pole occupies the other half.

The AC-AC or DC-AC/AC-DC macrocommutator magnetoelectrically-excited and reluctance in-wheel-hub motor/generator with different magnet-pole numbers on the wire-wound and unwound IPM inner stator can provide traction if the stator 5-phase armature-winding phase coils are sequentially energized. Here the unwound MSI outer rotor is made to *'chase'* sequentially switched wire-wound and unwound IPM inner stator magnet poles. A variation of this basic scheme employs a suitably magnetized disc on the shaft to directly sense the position of either the outer rotor itself.

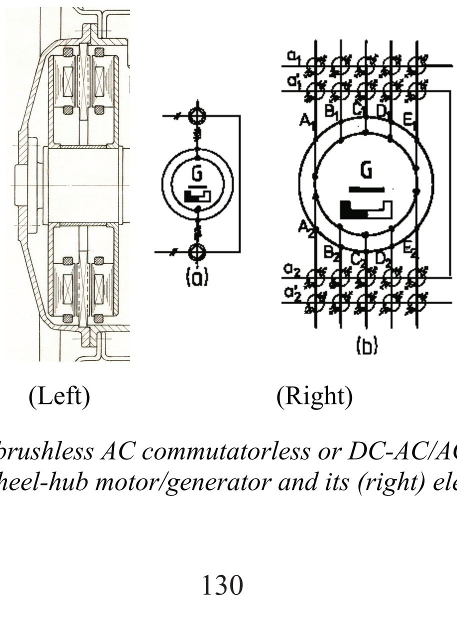
The AC-AC or DC-AC/AC-DC macrocommutator magnetoelectrically-excited and reluctance in-wheel-hub motor/generator adopts the *'IPM bias'* method, the IPM poles made of *Nd-Fe-B, Sm-Fe-Mo* or *Sm-Fe-Ti-B* rare-earth metal alloy is located at the centre of the inner-stator laminated-iron core.

Both the IPM poles and the 5-phase armature-winding phase coils on the inner stator create the magnetic field and the output torque is proportional to the square of the sum of both magnetic flux of the IPM inductor and the magnetic flux of the inner stator's 5-phase armature winding.

Significantly, the AC-AC or DC-AC/AC-DC macrocommutator sprocket-, tensioner- or road-wheel-hub motor/ generator is designed as a magnetoelectric and reluctance one. It is, indeed, both of these. Moreover, it is described as an IPM inductor (5-phase wire-wound and unwound IPM inner stator and unwound MSI outer rotor) sprocket-, tensioner- and road-wheel-hub motor/ generator.

C. Sprocket-, Tensioner- and Road Wheels (SM&GWs, TM&GWs and RM&GWs) with the AC-AC or DC-AC/AC-DC macrocommutator magnetoelectrically-excited in-wheel-hub motor/generator

As it is afore-mentioned, the core of the IMBT’s DBW AWD propulsion and BBW AWB dispulsion mechatronic control system are two SM&GWs and two TM&GWs as well as fourteen RM&GWs, which may incorporate a dissimilar brushless AC commutatorless or AC-DC/DC-AC macrocommutator [2 × 5-phase wire-wound (slotted-core) disc-shaped stators and unwound interior permanent magnet (IPM) rotor] magnetoelectrically-excited in-wheel-hub motor/ generator with much higher performance and greater compactness not only at the interconnection stage. The arrangement is very simple as shown in Figure 5.



(Left) (Right)

Fig. 5. The layout (left) of the brushless AC commutatorless or DC-AC/AC-DC macrocommutator IPM magnetoelectrically-excited in-wheel-hub motor/generator and its (right) electrical circuitry physical model

The axial-air-gap, unwound IPM disc-shaped rotor, 2 × 5-phase wire-wound (slotted-core) disc-shaped stators, high-power density (circa *0.1 kg/kW*), SM&GW or TM&GW as well as RM&GW AC commutatorless or AC-DC/DC-AC macrocommutator IPM magnetoelectrically-ecited in-wheel-hub motor/generator has been applied where high-driving and/or braking control is important, or where reduced in-wheel-hub motor/generator length is required.

The basic concept is shown in Figure 5. An in-wheel-hub motor/generator consisting of one rotor containing IPM poles centre between two stators is developed and installed for energy storage and retrevial mechanical power boosting.

This in-wheel-hub motor/generator eliminates the rotor magnetic-flux return ring (back iron), reduces overall electrical machine volume and rotor inertia, and eliminates the thrust bearing required to account for attractions between the rotor and stators. Here, the back iron is removed, and the magnetic-flux return path is through the two stators yokes.

4. Mathematical Model

In order to formulate a mathematical model of the sprocket-, tensioner- and/or road-wheel-hub motors/generators for the electrically-powered and mechatronically-controlled IMBT, analogous to the mathematical model of the brushed DC-AC/AC-DC commutator IPM magnetoelectrically-excited motor/generator with a rotating DC-AC/AC-DC mechanic-electrical commutator (in which phenomenon of current switching takes place as a function of the angular displacement of the rotor θ), taking into consideration equations of unholonomic constraints of the AC-AC, AC-DC-AC or DC-AC/AC-DC macrocommutator, a set of the second-order Euler-Lagrange differential equations of dynamics in a matrix notation for the AC commutatorless synchronous generator can be written. The dynamic behaviour of the AC commutatorless synchronous generator is considered, the activity and state system being formed from a set of the second-order Euler-Lagrange differential equations of dynamics. When an activity and state system is capable of dissipating kinetic co-energy is considered, then a set of the second-order Euler-Lagrange differential equations of dynamics in high-level system matrix notation may be expressed [5]:

$$\left[\frac{d}{dt}\left(\frac{\partial T^*}{\partial\|\dot{q}\|}\right)-\frac{\partial T^*}{\partial\|q\|}+\frac{\partial V}{\partial\|q\|}+\frac{d}{dt}\left(\frac{\partial T_F^*}{\partial\|\dot{q}\|}\right)-\|Q\|^T\right]\delta\|q\|=0\,, \tag{1}$$

where:

$$T^*\equiv\frac{1}{2}\|\dot{q}\|^T\|K(q,\dot{q})\|\|\dot{q}\|,\; V\equiv\frac{1}{2}\|q\|\|P(q,\dot{q})\|\|q\|,\; T_F^*\equiv\int_0^t\left(\frac{1}{2}\|\dot{q}\|^T\|N(q,\dot{q})\|\|\dot{q}\|+\frac{1}{2}sign\|\dot{q}\|^T\|\Delta Q\|\|\dot{q}\|\right)dt\,.$$

Taking into account above, the mathematical model of the AC commutatorless synchronous generator may be obtained by differentiating, respectively, the conservative kinetic co-energy and potential energy as well as dissipative kinetic co-energy so obtained with respect to generalised coordinates and velocities, respectively, as well as time to form a modified set of the second-order *Euler-Lagrange* differential equations of dynamics in high-level system matrix notation may be rewritten in the form:

$$\left[\frac{d}{dt}\left(\|\dot{q}\|^T\|K(q,\dot{q})\|\right)-\frac{1}{2}\|\dot{q}\|^T\frac{\partial\|K(q,\dot{q})\|}{\partial\|q\|}\|\dot{q}\|+\|q\|^T\|P(q,\dot{q})\|+\|\dot{q}\|\,\|N(q,\dot{q})\|+sign\|\dot{q}\|^T\|\Delta Q\|-\|Q\|^T\right]\delta\|q\|=0\,, \tag{2}$$

In a modified set of the second-order *Euler-Lagrange* differential equations of dynamics the regarded AC commutatorless synchronous generator, i.e., the activity state system decomposed on four activity and state hypo-systems: rotor mechanical hyposystem, interior-permanent-magnet (IPM) excitation electrical hyposystem and two wounded armature electrical hypo-systems, the particular high-level system vectors and matrices incorporating three low-level system hypo-vectors and hypo-matrices:

$$\|q\| = \left\|\left[q^m\right]^T\left[q^e\right]^T\right\|^T,\ \|\dot{q}\| = \left\|\left[\dot{q}^m\right]^T\left[\dot{q}^e\right]^T\right\|^T,\ \|Q\| = \left\|\left[Q^m\right]^T\left[Q^e\right]^T\right\|^T,\ \|\Delta Q\| = diag\left\|\left[\Delta Q^m\right]\ \left[\Delta Q^e\right]\right\|,$$

$$\|K(q,\dot{q})\| = \begin{Vmatrix}\left[K^m(q,\dot{q})\right] & [0] \\ [0] & \left[K^e(q,\dot{q}\right]\end{Vmatrix},\ \|P(q,\dot{q})\| = \|0\|,\ \|N(q,\dot{q})\| = \begin{Vmatrix}\left[N^m(q,\dot{q})\right] & [0] \\ [0] & \left[N^e(q,\dot{q})\right]\end{Vmatrix},$$

where for the particular low-level system hypo-vectors and hypo-matrices incorporating twelve low-level system scalars of generalized motion coordinates; rotation moments (moments of forces or torques); rotational inertia (moments of inertia); rotational stiffnesses; rotational resistances (due to the viscous friction); as well as rotation moment drops (due to the coulomb friction) are represented as follows:

$$\left[q^m\right] = \theta,\ \left[\dot{q}^m\right] = \dot{\theta} = \omega,\ \left[q^e\right] = \left[\left(q^{f_0}\right)\quad\left(q^{a_1}\right)\quad\left(q^{a_2}\right)\right]^T,\ \left(q^{\,f\,0}\right) = q^{\,f\,0},$$

$$\left(q^{a1}\right) = \left(q^{A_1}\quad q^{B_1}\quad q^{C_1}\quad q^{D_1}\quad q^{E_1}\right)^T,\ \left(q^{a2}\right) = \left(q^{A_2}\quad q^{B_2}\quad q^{C_2}\quad q^{D_2}\quad q^{E_2}\right)^T$$

$$\left(\dot{q}^{f\,0}\right) = i^{fo} = i_{f\,0},$$

$$\left(\dot{q}^{a_1}\right) = \left(\dot{q}^{A_1}\quad \dot{q}^{B_1}\quad \dot{q}^{C_1}\quad \dot{q}^{D_1}\quad \dot{q}^{E_1}\right)^T = \left(i^{A_1}\quad i^{B_1}\quad i^{C_1}\quad i^{D_1}\quad i^{E_1}\right)^T,$$

$$\left(\dot{q}^{a_2}\right) = \left(\dot{q}^{A_2}\quad \dot{q}^{B_2}\quad \dot{q}^{C_2}\quad \dot{q}^{D_2}\quad \dot{q}^{E_2}\right)^T = \left(i^{A_2}\quad i^{B_2}\quad i^{C_2}\quad i^{D_2}\quad i^{E_2}\right)^T,$$

$$\left[K^m(q,\dot{q})\right] = J\,,$$

$$\left[K^e(q,\dot{q})\right] = \begin{bmatrix} L^f & L^{fA}(\theta) & L^{fB}(\theta) & L^{fC}(\theta) & L^{fD}(\theta) & L^{fE}(\theta) & L^{fF}(\theta) \\ L^{Af}(\theta) & L^{A}(\theta) & L^{AB}(\theta) & L^{AC}(\theta) & 0 & 0 & 0 \\ L^{Bf}(\theta) & L^{BA}(\theta) & L^{B}(\theta) & L^{BC}(\theta) & 0 & 0 & 0 \\ L^{Cf}(\theta) & L^{CA}(\theta) & L^{CB}(\theta) & L^{C}(\theta) & 0 & 0 & 0 \\ L^{Df}(\theta) & 0 & 0 & 0 & L^{D}(\theta) & L^{DE}(\theta) & L^{DF}(\theta) \\ L^{Ef}(\theta) & 0 & 0 & 0 & L^{ED}(\theta) & L^{E}(\theta) & L^{EF}(\theta) \\ L^{Ff}(\theta) & 0 & 0 & 0 & L^{FD}(\theta) & L^{FE}(\theta) & L^{F}(\theta) \end{bmatrix},$$

$$\left[N^m(q,\dot{q})\right] = D,$$

$$\left[N^e(q,\dot{q})\right] = diag\left[0\ \left(R^{a_1}\right)\ \left(R^{a_2}\right)\right] = diag\left[0\ \left(R^{A_1}\quad R^{B_1}\quad R^{C_1}\quad R^{D_1}\quad R^{E_1}\right)\ \left(R^{A_2}\quad R^{B_2}\quad R^{C_2}\quad R^{E_2}\quad R^{E_2}\right)\right],$$

$$\left[\Delta Q^m\right] = \Delta M,$$

$$\left[\Delta Q^{e}\right]=diag\left[0\ \left(\Delta Q^{a_1}\right)\ \left(\Delta Q^{a_2}\right)\right]=diag\left[0\ \left(\Delta Q^{A_1}\ \ \Delta Q^{B_1}\ \ \Delta Q^{C_1}\ \ \Delta Q^{D_1}\ \ \Delta Q^{E_1}\right)\ \left(\Delta Q^{A_2}\ \ \Delta Q^{B_2}\ \ \Delta Q^{C_2}\ \ \Delta Q^{D_2}\ \ \Delta Q^{E_2}\right)\right],$$

$$\left[Q^{m}\right]=M,$$

$$\left[Q^{e}\right]=\left[0\ \left(Q^{a_1}\right)\ \left(Q^{a_2}\right)\right]^{T}=\left[0\ \left(Q^{A_1}\ \ Q^{B_1}\ \ Q^{C_1}\ \ Q^{D_1}\ \ Q^{E_2}\right)\ \left(Q^{A_2}\ \ Q^{B_2}\ \ Q^{C_2}\ \ Q^{D_2}\ \ Q^{E_2}\right)\right]^{T},$$

The mathematical model of the AC commutatorless synchronous generator, in the form of a set of the second-order *Euler-Lagrange* differential equations of dynamics in low-level system matrix notation (2), in minimal mechanical and electrical co-ordinates assumed for analysis of the regarded activity and state system, may be formulated, using a physical model of the AC commutatorless synchronous generator shown in Figure 5.

$$\left[\frac{d}{dt}\left(\left\|\begin{matrix}\left[\dot{q}^{m}\right]\\ \left[\dot{q}^{f_0}\right]\\ \left[\dot{q}^{a_1}\right]\\ \left[\dot{q}^{a_2}\right]\end{matrix}\right\|^{T}\left\|\begin{matrix}\left[K^{m}(q,\dot{q})\right] & [0] & [0] & [0]\\ [0] & \left[K^{f_0}(q,\dot{q})\right] & \left[K^{f_0a_1}(q,\dot{q})\right] & \left[K^{f_0a_2}(q,\dot{q})\right]\\ [0] & \left[K^{a_1f_0}(q,\dot{q})\right] & \left[K^{a_1}(q,\dot{q})\right] & [0]\\ [0] & \left[K^{a_2f_0}(q,\dot{q})\right] & [0] & \left[K^{a_2}(q,\dot{q})\right]\end{matrix}\right\|\right)-\right.$$

$$-\frac{1}{2}\left\|\begin{matrix}\left[\dot{q}^{m}\right]\\ \left[\dot{q}^{f_0}\right]\\ \left[\dot{q}^{a_1}\right]\\ \left[\dot{q}^{a_2}\right]\end{matrix}\right\|^{T}\left\|\begin{matrix}\left[K^{m}(q,\dot{q})\right] & [0] & [0] & [0]\\ [0] & \left[K^{f_0}\right] & \left[K^{f_0a_1}(q,\dot{q})\right] & \left[K^{f_0a_2}(q,\dot{q})\right]\\ [0] & \left[K^{a_1f_0}(q,\dot{q})\right] & \left[K^{a_1}(q,\dot{q})\right] & [0]\\ [0] & \left[K^{a_2f_0}(q,\dot{q})\right] & [0] & \left[K^{a_2}(q,\dot{q})\right]\end{matrix}\right\|\left\|\begin{matrix}\left[\dot{q}^{m}\right]\\ \left[\dot{q}^{f_0}\right]\\ \left[\dot{q}^{a_1}\right]\\ \left[\dot{q}^{a_2}\right]\end{matrix}\right\|+$$

$$\left.+\left\|\begin{matrix}\left[\dot{q}^{m}\right]\\ \left[\dot{q}^{f_0}\right]\\ \left[\dot{q}^{a_1}\right]\\ \left[\dot{q}^{a_2}\right]\end{matrix}\right\|^{T}\left\|\begin{matrix}\left[N^{m}\right] & [0] & [00] & [0]\\ [0] & [0] & [0] & [0]\\ [0] & [0] & \left[N^{a_1}\right] & [0]\\ \{0\} & [0] & [0] & \left\{N^{a_2}\right\}\end{matrix}\right\|+sign\left\|\begin{matrix}\left[\dot{q}_{m}\right]\\ \left[\dot{q}_{f_0}\right]\\ \left[\dot{q}_{a_1}\right]\\ \left[\dot{q}_{a_2}\right]\end{matrix}\right\|^{T}\left\|\begin{matrix}\left[\Delta Q^{m}\right] & [0] & [0] & [0]\\ [0] & [0] & [0] & [0]\\ [0] & [0] & \left[\Delta Q^{a_1}\right] & [0]\\ [0] & [0] & [0] & \left[\Delta Q^{a_2}\right]\end{matrix}\right\|-\left\|\begin{matrix}\left[Q^{m}\right]\\ [0]\\ \left[Q^{a_1}\right]\\ \left[Q^{a_2}\right]\end{matrix}\right\|^{T}\right]\delta\left\|\begin{matrix}\left[q^{m}\right]\\ \left[q^{f_0}\right]\\ \left[q^{a_1}\right]\\ \left[q^{a_2}\right]\end{matrix}\right\|=0.\quad(3)$$

For the aforementioned case, in equations of unholonomic constraints the hypo-matrices of voltage and current commutating nodes (points of junction) '*lK*' and '*Kl'*, respectively:

$$\left\|V_{sP}\right\|=\left\|C^{Ps}\right\|^{T},\tag{4}$$

for instance, can be represented in the following form:

$$\left\|C^{Ps}\right\|=diag\left\|\ [I]\ \ [I]\ \ \left[C^{a_1}\right]\ \ \left[C^{a_2}\right]\right\|,\tag{5}$$

$$\left[C^{a_1}\right]=\begin{bmatrix}C^{a_1A_1} & C^{a_1B_1} & C^{a_1C_1} & C^{a_1D_1} & C^{a_1E_1}\\ C^{a_1'A_1} & C^{a_1'B_1} & C^{a_1'C_1} & C^{a_1'D_1} & C^{a_1'E_1}\end{bmatrix};\ \left[C^{a_2}\right]=\begin{bmatrix}C^{a_2A_2} & C^{2B_2} & C^{a_2C_2} & C^{a_2D_2} & C^{a_2E_2}\\ C^{a_2A_2} & C^{a_2B_2} & C^{a_2'C_2} & C^{a_{21}D_2} & C^{a_2E_2}\end{bmatrix},$$

$$[I]=1,$$

$$\left[C^{a_1}\right]=\left[C^{Kl}\left[(\theta-\alpha)+(n-1)\frac{2\pi}{m}\right]\right],\qquad \left[C^{a_2}\right]=\left[C^{Kl}\left[(\theta-\alpha)+(n-1)\frac{2\pi}{m}+\frac{\pi}{m}\right]\right],\tag{6}$$

K = A, B, C, D, E; l = a, a'; n = 1, 2, 3; m = 3.

where: α - control angular displacement, adequate to the triggering-on angle of the AC-DC macrocommutator's controlled electrical valves.

Yet, current commutating nodes in the equation (6), after taking into consideration the difference of angular displacements $\theta - \alpha$, for instance, for the macrocommutator's ideal controlled electrical valves, can be expressed in the following form:

$$C^{Kl} = \begin{cases} \pm 1 \to i^K & for\left(\theta - \alpha\right) \\ 0 \to i^K & for\left(\theta - \alpha\right). \\ \mp 1 \to i^K & for\left(\theta - \alpha\right) \end{cases} \tag{7}$$

After taking into account the equations of unholonomic constraints of the MCT or MOSFER AC-DC/DC-AC macrocommutator in the differential equations of dynamics (1) and (2) one can be obtained differential equations of dynamics (8), establishing of a mathematical model of the brushless AC-DC/DC-AC macrocommutator magnetoelectrically-excited in-wheel-hub generator/ motor with the MCT or MOSFET AC-DC/DC-AC macrocommutator, the first acting as the electrical machine's MCT or MOSFET AC-DC rectifier/DC-AC inverter with two five-input times two-output ASIM in the double heteropolar commutating group (full-bridge) connection:

$$\left[\frac{d}{dt}\left(\begin{Vmatrix} [\dot{q}^{m}] \\ [\dot{q}^{f_0}] \\ [\dot{q}^{a_1}] \\ [\dot{q}^{a_3}] \end{Vmatrix}^T \begin{Vmatrix} [K_m(\alpha)] & [0] & [0] & [0] \\ [0] & [K_{f_0}] & [K_{f_0 a_1}(\alpha)] & [K_{f_0 a_2}(\alpha)] \\ [0] & [K_{a_1 f_0}(\alpha)] & [K_{a_1}(\alpha)] & [0] \\ [0] & [K_{a_2 f_0}(\alpha)] & [0] & [K_{a_2}(\alpha)] \end{Vmatrix} \right) + \begin{Vmatrix} [\dot{q}_{m}] \\ [\dot{q}_{f_0}] \\ [\dot{q}_{a_1}] \\ [\dot{q}_{a_3}] \end{Vmatrix}^T \begin{Vmatrix} [0] & [0] & [0] & [0] \\ [0] & [0] & [K^*_{f_f a_1}(\alpha)] & [K^*_{f_0 a_2}(\alpha)] \\ [0] & [K^*_{a_1 f_0}](\alpha) & [K^*_{a_1}(\alpha)] & [0] \\ [0] & [K^*_{a_2 f_0}(\alpha)] & [0] & [K^*_{a_2}(\alpha)] \end{Vmatrix} \| [\dot{q}_m] \| + \right.$$

$$+ \begin{Vmatrix} [\dot{q}_{m}] \\ [\dot{q}_{f_0}] \\ [\dot{q}_{a_1}] \\ [\dot{q}_{a_2}] \end{Vmatrix}^T \begin{Vmatrix} [0] & [0] & [0] & [0] \\ [0] & [0] & [0] & [0] \\ [0] & [K^{**}_{a_1 f_0}(\alpha)] & [K^{**}_{a_1}(\alpha)] & [0] \\ [0] & [K^{**}_{a_2 f_0}(\alpha)] & [0] & [K^{**}_{a_2}(\alpha)] \end{Vmatrix} \| [\dot{q}_m] - [\dot{q}_c] \| - \frac{1}{2} \begin{Vmatrix} [\dot{q}_{m}] \\ [\dot{q}_{f_0}] \\ [\dot{q}_{a_1}] \\ [\dot{q}_{a_2}] \end{Vmatrix}^T \begin{Vmatrix} [0] & [0] & [0] & [0] \\ [0] & [0] & [0] & [0] \\ [0] & [K^{*}_{a_1 f_0}(\alpha)] & [K^{*}_{a_1}(\alpha)] & [0] \\ [0] & [K^{*}_{a_2 f_0}(\alpha)] & [0] & [K^{*}_{a_2}(\alpha)] \end{Vmatrix} \begin{Vmatrix} [q'_{m}] \\ [\dot{q}_{f_0}] \\ [\dot{q}_{a_1}] \\ [\dot{q}_{a_2}] \end{Vmatrix} +$$

$$\left. + \begin{Vmatrix} [\ddot{q}_{m}] \\ [\dot{q}_{f_0}] \\ [\dot{q}_{a_1}] \\ [\dot{q}_{a_2}] \end{Vmatrix}^T \begin{Vmatrix} [N_m] & [0] & [0] & [0] \\ [0] & [0] & [0] & [0] \\ [0] & [0] & [N_{a_1}] & [0] \\ [0] & [0] & [0] & [N_{a}] \end{Vmatrix} + sign \begin{Vmatrix} [\dot{q}_{m}] \\ [\dot{q}_{f_0}] \\ [\dot{q}_{a_1}] \\ [\dot{q}_{a_2}] \end{Vmatrix}^T \begin{Vmatrix} [\Delta Q_m] & [0] & [0] & [0] \\ [0] & [0] & [0] & [0] \\ [0] & [0] & [\Delta Q_{a_1}] & [0] \\ [0] & [0] & [0] & [\Delta Q_{a_2}] \end{Vmatrix} - \begin{Vmatrix} [Q_{m}] \\ [0] \\ [Q_{a_1}] \\ [Q_{a_2}] \end{Vmatrix}^T \right] \delta \begin{Vmatrix} [q_{m}] \\ [q_{f_0}] \\ [q_{a_1}] \\ [q_{a_2}] \end{Vmatrix} = 0\,, \tag{8}$$

where hypovectors $\left[Q_{a_1}\right] = \left[Q^{a_1}\right] = \left[Q^{s_1}\right]$; $\left[Q_{a_2}\right] = \left[Q^{a_2}\right] = \left[Q^{s_2}\right]$ and $\left[q_{a_1}\right] = \left[q_{s_1}\right]$; $\left[q_{a_2}\right] = \left[q_{s_2}\right]$ are defined by the relationships (4), (5) and (6), however, the remaining hypomatrices are expressed by the following forms:

$$\left[N_{a_1}\right] = \left[C^{a_1}\right]^T \left[N^{a_1}\right];\ \left[N_{a_2}\right] = \left[C^{a_2}\right]^T \left[N^{a_2}\right], \tag{9}$$

$$\left[\Delta Q_{a_1}\right] = \left[C^{a_1}\right]^T \left[\Delta Q^{a_1}\right];\ \left[\Delta Q_{a_2}\right] = \left[C^{a_2}\right]^T \left[\Delta Q^{a_2}\right], \tag{10}$$

$$\left[K_{a_1}(\alpha)\right] = \left[C^{a_1}\right]^T \left[K^{a_1}(\theta)\right] \left[C^{a_1}\right];\ \left[K_{a_2}(\alpha)\right] = \left[C^{a_2}\right]^T \left[K^{a_2}(\theta)\right] \left[C^{a2}\right], \tag{11}$$

$$\left[K_{f_0a_1}(\alpha)\right]=\left[K_{a_1f_1}(\alpha)\right]^T=\left[C^{a_1}\right]^T\left[K^{f_0a_1}(\theta)\right];\quad \left[K_{f_0a_2}(\alpha)\right]=\left[C^{a_2}\right]^T\left[K^{f_0a_2}(\theta)\right], \tag{12}$$

$$\left[K_{f_0a_1}\alpha\right]=\left[K_{a_1f_0}(\alpha)\right]^T=\left[C^{a_1}\right]^T\left[K^{f_0a_1}(\theta)\right];\quad \left[K_{f_0a_2}(\alpha)\right]=\left[K_{a_2}(\alpha)\right]^T=\left[C^{a_2}\right]^T\left[K^{f_0a_2}\theta\right], \tag{13}$$

$$\left[K^*_{f_0a_1}(\alpha)\right]=\left[K^*_{a_1f_0}(\alpha)\right]^T=\left[C^{a_1}\right]^T\left[K_*^{f_0a_1}(\theta)\right]=\left[C^{a_1}\right]^T\frac{\partial\left[K^{f_0a_1}(\theta)\right]}{\partial\theta}, \tag{14}$$

$$\left[K^*_{f_0a_2}(\alpha)\right]=\left[K^*_{a_2f_0}(\alpha)\right]^T=\left[C^{a_2}\right]^T\left[K_*^{f_0a_2}(\theta)\right]=\left[C^{a_2}\right]^T\frac{\partial\left[K_*^{f_0a_2}(\theta)\right]}{\partial\theta}, \tag{15}$$

$$\left[K^{**}_{a_1}(\alpha)\right]=\left[C^{a_1}\right]^T\left[K^{a_1}(\theta)\right]\frac{\partial\left[C^{a_1}\right]}{\partial\alpha}\;;\;\left[K^{**}_{a_2}(\alpha)\right]=\left[C^{a_2}\right]^T\left[K^{a_2}(\theta)\right]\frac{\partial\left[C^{a_2}\right]}{\partial\alpha}, \tag{16}$$

$$\left[K^{**}_{f_0a_1}(\alpha)\right]=\left[K^{f_0a_1}(\theta)\right]\frac{\partial\left[C^{a_1}\right]}{\partial\alpha}\;;\;\left[K^{**}_{f_0a_2}(\alpha)\right]=\left[K_{f_0a_2}(\theta)\right]\frac{\partial\left[C^{a_2}\right]}{\partial\alpha}, \tag{17}$$

$$[q_m]=\theta\;;\;[q_{f_0}]=i_{f_0}\;;\;[q_{a_1}]=[q_{a_1}\quad q_{a_1'}]^T\;;\;[q_{a_2}]=[q_{a_2}\quad q_{a_2'}]^T\;;\;[q_c]=\alpha, \tag{18}$$

$$[\dot q_m]=\dot\theta=\omega\;;\;[\dot q_{f_0}]=i_{f_0}\;;\;[\dot q_{a_1}]=[i_{a_1}\quad i_{a_1'}]^T\;;\;[\dot q_{a_2}]=[i_{a_2}\quad i_{a_2'}]^T\;;\;[\dot q_c]=\nu, \tag{19}$$

$$\|\dot q\|=\|\dot q^P\|=\|C^{Ps}\|\,\|\dot q_s\|=\begin{Vmatrix}[I] & [0] & [0] & [0]\\ [0] & [I] & [0] & [0]\\ [0] & [0] & [C^{a_1}] & [0]\\ [0] & [0] & [0] & [C^{a_2}]\end{Vmatrix}\begin{Vmatrix}[\dot q_m]\\ [\dot q_{f_0}]\\ [\dot q_{a_1}]\\ [\dot q_{a_2}]\end{Vmatrix}, \tag{20}$$

$$\delta\|q\|=\delta\|q^P\|=\|C^{Ps}\|\,\delta\|q_s\|=\begin{Vmatrix}[I] & [0] & [0] & [0]\\ [0] & [I] & [0] & [0]\\ [0] & [0] & [C^{a_1}] & [0]\\ [0] & [0] & [0] & [C^{a_2}]\end{Vmatrix}\delta\begin{Vmatrix}[q_m]\\ [q_{f_0}]\\ [q_{a_1}]\\ [q_{a_2}]\end{Vmatrix}, \tag{21}$$

as well as

$$U_{a_1}=U_{a_1}-U_{a_1'}\;;\;U_{a_2}=U_{a_2}-U_{a_2'}\;;\;i_{a_1}=i_{a_1'}\;;\;i_{a_2}=i_{a_2'}\;;\;d\theta/dt=\omega\;;\;d\alpha/dt=\nu.$$

Relationships (9) - (20) define the hypo-matrices of resistances, self- and mutual-inductances, rotation coefficients of self- and mutual-inductances, and commutation coefficients of self- and mutual-inductances of the differential equations of dynamics (8), consisting the mathematical model of the brushless AC-DC/DC-AC macrocommutator magnetoelectrically-excited in-wheel-hub generator/starter motor with the MCT or MOSFET AC-DC/DC-AC macrocommutator with the five-input times two-output ASIM in the double heteropolar commutating group (full-bridge) connection. For the given control angular displacement α, the elements of the hypo-matrices (9) - (20) are dependent from the sprocket-, tensioner- or road-wheel hub, i.e., the generator/motor outside rotor angular displacement θ.

5. Conclusions

The novel brushless AC-DC/DC-AC & AC-AC macrocommutator IPM magnetoelectrically-excited in-wheel-hub generator/motor is a special electrical machine for generating and distributing electrical energy and driving or braking in the IMBT. This innovative high-tech optimises electrical energy utilization and decreases specific fuel consumption (SFC) and thus pollutant emissions - radically.

From the mathematical model of the brushless AC-DC/AC-DC macrocommutator magneto-electrically-excited in-wheel-hub generator/motor, establishing a set of the second order *Euler-Lagrange* differential equations of dynamics with the unholonomic constraints results that these equations give mutual relationship amongst the generalized coordinates and generalized velocities as well as generalized quasi-velocities and generalized excitation forces, as well as the parameters and quasi-parameters of the electrical machine.

On the basis of a set of the second-order *Euler-Lagrange* differential equations of dynamics in a low-level system matrix notation, establishing the mathematical model, using simulation computer program, for instance, PSPICE may be provided computerized analytical studies of the brushless AC-DC/DC-AC macrocommutator magnetoelectrically-excited in-wheel-hub generator/ motor. In this manner, the above formulated mathematical model and its computerized analytical studies may make possible simulation of the dynamic and static mechanic-electrical or electro-mechanical processes.

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